

Depth-Limited Significant Wave Height: A Spectral Approach

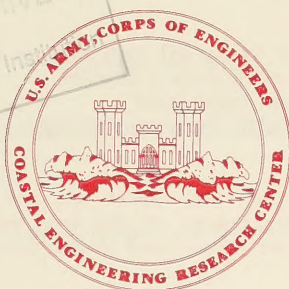
by

C. Linwood Vincent

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PREFACE

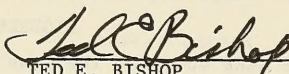
This report presents a method for estimating depth-limited significant wave height of an irregular wave field. The work was carried out under the U.S. Army Coastal Engineering Research Center's (CERC) Wave Estimation for Design work unit, Coastal Flooding and Storm Protection Program, Coastal Engineering Area of Civil Works Research and Development.

The report was prepared by Dr. C. Linwood Vincent, Chief, Coastal Oceanography Branch, under the general supervision of Mr. R.P. Savage, Chief, Research Division. J.E. McTamany prepared the computer integration scheme; W.N. Seelig and L.L. Broderick provided laboratory data.

Technical Director of CERC was Dr. Robert W. Whalin, P.E., upon publication of this report.

Comments on this publication are invited.

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TED E. BISHOP

Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins ¹

¹To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

E	total variance in wind sea, often called energy
$E(f)$	variance density, often called energy density
E_h	depth-limited value of total variance
$E_m(f)$	upper bound on energy density in a frequency, f
F	variance density spectrum in wave number space
f	frequency
f_c	low-frequency cutoff
f_p	peak frequency of the spectrum
H	depth-controlled wave height (spectral)
H_d	depth-limited wave height (monochromatic)
H_ℓ	depth-limited wave height (irregular sea)
H_{Mo}	zero-moment wave height, also called significant wave height
H_{max}	largest individual wave
$H_{1/3}$	significant wave height
h	depth
k	wave number
R	transcendental function of dimensionless frequency ω_h
U	windspeed
α	Phillips' equilibrium coefficient
π	3.1415
Φ	dimensionless function describing deviation from deepwater equilibrium range
ω_h	dimensionless combination of g , f , and h

by
C. Limwood Vincent

I. INTRODUCTION

Research into the shape of wind wave spectra in finite-depth water has suggested an expression for the upper limit on the energy density as a function of depth and frequency (Kitaigorodskii, Krasitskii, and Zaslavskii, 1975). In this report this expression is integrated over the part of the spectrum expected to contain energy to estimate a limit on the energy, E , in the wind wave spectrum and to define a depth-limited significant wave height, H_ℓ :

$$H_\ell = 4.0(E)^{1/2} \quad (1)$$

More precisely, the quantity estimated is the variance of the sea surface to which E is directly related. Following convention, E and $E(f)$ denote energy and energy density spectrum although the true units of computation are length squared and length squared per hertz. The term zero-moment wave height, H_{M0} will be used to denote $4.0(E)^{1/2}$. $H_{1/3}$ is the average height of the one-third highest waves. H_ℓ denotes values of H_{M0} that are depth limited. In deep water, H_{M0} is approximately $H_{1/3}$, but this is not necessarily true in shallow depths. H_d refers to the depth-limited monochromatic wave. The variation of H_ℓ with depth, h , is investigated and compared with the monochromatically derived depth-limited wave height, H_d . Because H_{M0} and $H_{1/3}$ are about equal in deep water, they are both frequently called significant wave height.

This report briefly reviews the theoretical development of the limiting form for spectral densities as a function of water depth and presents field evidence supporting this form. The simple derivation of the depth-limited energy and significant wave height is then given, followed by field and laboratory data evaluating the prediction equation. Unless otherwise noted, the developments of this report are restricted to wave conditions described by a wave spectrum of some width such as an active wind sea or a decaying sea.

II. THEORETICAL BACKGROUND

Phillips (1958) suggested that there should be a region of the spectrum of wind-generated gravity waves in which the energy is limited by wave steepness. Phillips derived an expression for the limiting density in deep water:

$$E_m(f) = \alpha g^2 f^{-5} (2\pi)^{-4} \quad (2)$$

where α was considered to be a universal constant. Field studies reviewed by Plant (1980) demonstrated that equation (2) adequately describes the part of the wind sea spectrum above the peak frequency of the spectrum. However, Hasselmann, et al. (1973) indicated that the equilibrium coefficient α is not constant but varies systematically with wave growth leading the authors to speculate that resonant interactions in the spectrum force the spectrum to evolve to the form of equation (2). Toba (1973) suggested that the equilibrium range form might be proportional to $U_* f^{-4}$ in order to remove the variation of α .

Kitaigorodskii, Krasitskii, and Zaslavskii (1975), using Phillips' (1958) expression for the steepness limited form of a wave spectrum, F , in terms of the wave number modulus,

$$F(k) = k^{-3} \quad (3)$$

solved the transformation of equation (3) to a frequency spectrum in finite-depth water. The finite-depth form, $E_m(f, h)$, was shown to be equal to the deepwater form (eq. 2) times a dimensionless function, $\phi(\omega_h)$,

$$E_m(f, h) = \frac{\alpha g^2 f^{-5}}{(2\pi)^4} \phi(\omega_h) \quad (4)$$

Kitaigorodskii, Krasitskii, and Zaslavskii suggested a value of 0.0081 for α .

The function ϕ requires an iterative procedure for solution and is defined as

$$\phi(\omega_h) = R^{-2}(\omega_h) \left[1 + \frac{2\omega_h^2 R(\omega_h)}{\sinh(2\omega_h^2 R(\omega_h))} \right]^{-1} \quad (5)$$

$$\text{with } \omega_h = \omega(h/g)^{1/2} \quad (6)$$

where $\omega = 2\pi f$ and $R(\omega_h)$ is obtained from the solution of

$$R(\omega_h) \tanh\left(\omega_h^2 R(\omega_h)\right) = 1 \quad (7)$$

The dimensionless parameter ω_h related the frequency and depth to the deviation from the deepwater form. When ω_h is greater than 2.5, ϕ is approximately 1, when ω_h is zero ϕ is zero. When ω_h is less than 1

$$\phi(\omega_h) \cong \omega_h^2/2 \quad (8)$$

For ω_h less than 1, a combination of equations (8) and (4) leads to the expression

$$E_m(f, h) = \alpha g h f^{-3} / (2(2\pi)^2) \quad (9)$$

Thus in the shallow-water limit, the bound on energy density in the wave spectrum is proportional to f^{-3} compared to f^{-5} in deep water, and depth is included linearly.

Resio and Tracy (U.S. Army Engineer Waterways Experiment Station, personal communication, 1981) have analyzed the resonant interactions and derived equivalent expressions to equations (3) and (4) on the basis of similarity theory. The conclusion of their theoretical study is that the role of the wave-wave interactions in both deep and shallow water is to force the spectrum to evolve to the form of equation (4). Their theory may be distinguished from

that of Kitaigorodskii, Krasitskii, and Zaslavskii (1975) in that their coefficient α is expected to vary with wave conditions and not remain a universal constant.

III. FIELD EVIDENCE FOR THE FINITE-DEPTH SPECTRAL FORM

Prior to Kitaigorodskii, Krasitskii, and Zaslavskii (1975), Kakimura (1967) and Druat, Massel, and Zeidler (1969) had noted that the shape of the spectrum in shallow water deviated from Phillips' (1958) form. Kitaigorodskii, Krasitskii, and Zaslavskii cited evidence supporting the f^{-3} form, as did Thornton (1977) and Gadzhiiyev and Kratsitsky (1978). Ou (1980) provided laboratory evidence for equation (4). A review of spectra collected at the Coastal Engineering Research Center's (CERC) Field Research Facility (FRF) at Duck, North Carolina, and at other gages in shallow water supports a near f^{-3} spectral slope in depths less than 10 meters for large wave energies.

These findings indicate a further evaluation is needed of how well the equation fits observed spectra. During the Atlantic Remote Sensing Land and Ocean Experiment (ARSLOE) conducted in October and November 1980 at the FRF, North Carolina, wave spectra were collected in 36 meters of water about 36 kilometers offshore of the CERC facility (Fig. 1), using the National Ocean Survey's directional buoy, XERB, with accelerometer buoys in depths of 25, 18, and 17 meters of water located at distances of 12, 6, and 3 kilometers offshore along a line from the facility to the XERB. In addition, data from Baylor gages at seven locations in 1.5- to 9-meter depths along the FRF pier were collected. On 25 October 1980 a large, low-pressure system generated waves with significant heights up to 5.0 meters. Data were collected continuously at the XERB during the period of high waves and spectra at all gages were computed every 20 minutes.

As a test the observed spectra, $E(f)$, were normalized to the following forms

$$\alpha_p(f) = f^5 E(f) (2\pi)^{-4} / g^2 \quad (10)$$

$$\alpha_h(f) = f^5 E(f) (2\pi)^4 / g^2 \Phi(\omega_h) \quad (11)$$

$$\alpha_3(f) = f^3 E(f) / g_h \quad (12)$$

Equation (10) is an estimate of the equilibrium coefficient as a function of frequency if the spectra follow the deepwater form. Likewise, equation (11) is an estimate of the coefficient if the spectra follow the proposed finite-depth form, and equation (12) is an estimate of the coefficient if the proposed shallow water (ω_h less than 1) holds over most of the spectrum. If any of these forms fit a spectrum then the corresponding function $\alpha(f)$ should be constant with frequency. Therefore in a regression of f against $\alpha(f)$, f should explain no variance; consequently, the degree of fit to the spectrum by each of the three forms can be estimated by how poorly f explains variance in the regression and how flat the slope with f is. The regressions were performed over the region from the spectral peak to twice the spectral peak and the results are tabulated in Tables 1 and 2.

Table 1. Normalized form regression (analysis average of percent variance¹ in regression of normalized form against frequency, f , explained by f).

Gage	Mean depth (m)	Deepwater form ²	Finite-depth form ³	Shallow-water limit form ⁴
XERB	36	43	17	48
710	25	53	28	8
630	18	40	13	10
620	17	55	24	18
625	9	68	35	32
655	5	70	60	59
615	2	72	60	59

¹Since the proposed form is supposed to remove variation with f , a high explained variance with f indicates that the form does not fit the spectra well.

$$^2 f^5 E(f) (2\pi)^{-4} / g^2$$

$$^3 f^5 E(f) (2\pi)^{-4} / g^2 \phi(\omega_h)$$

$$^4 f^3 E(f) / g_h$$

Table 2. Average slope, $\times 10^{-3}$, against f^1 .

Gage	$\alpha_p(f)$	$\alpha_h(f)$	$\alpha_3(f)$
XERB	31	17	10
710	54	35	4
630	47	18	-3
620	49	25	3
625	82	81	35
655	60	101	53
615	18	67	33

¹Slope α is unit change of α per hertz. Region of the spectrum analyzed in regression analysis is about 0.1 hertz.

Using the criteria established above, data summarized in Table 1 indicate that in all cases either the finite-depth form or the f^{-3} limit appears to fit the spectra better than the deepwater form. This is because f consistently explains less variance in these regressions than in the regressions against the deepwater form. In a regression analysis under an assumption of normally distributed variates, the hypothesis of zero correlation is rejected for the number of frequency components from f_p to $2f_p$ if the regression coefficient is greater than 0.632 at a 5 percent level of significance. This translates to a value of 40 percent for the values in Table 2. Table 1 indicates that the average R^2 for the regressions in the deepwater form are always greater than 40 percent, suggesting that there is correlation with f . The average finite-depth form value is less than 40 percent for all but two (655 and 615) of the gages, suggesting a tendency for no correlation with f . The shallow-water limit results suggest zero correlation except for gages XERB, 655, and 615. Table 2 indicates that the slopes are, in general, lower as well. Plots of $f^3 E(f)$ and $f^5 E(f)$ show that the spectra appear to more closely follow a f^{-3} slope (Fig. 2).

The results of the regression analysis for the gages at depths greater than 9 meters appear to be more closely fit by a f^{-3} form than the results at 9 meters and at shallower gages. The observed spectra at the shallower gages tend to be less than the proposed upper limit. It is thought that refraction, bottom friction, and massive breaking must dominate the spectra in and around the peak, suppressing the values below the proposed limiting value. This would indicate that in very shallow water, the proposed form may be conservative. Plots of storm spectra at different gage sites are compared to the limiting form in Figure 3.

The variation of the equilibrium coefficient α computed over the range f_p to $2f_p$ varies based on gage and time (as represented by sea and swell conditions), with α for the sea conditions being larger. Additionally, there appeared to be a tendency for α to increase slightly from deep to shallow water. On occasion α calculated at the peak of the spectrum exceeded the value of 0.0081. However, when the α value at the peak was compared to the α value averaged over the frequencies from f_p to $2f_p$, it was evident that the average value was much less than the value at the peak.

The field evidence from a variety of sources supports the conclusion that the maximum energy densities above the peak frequency of the spectrum can be approximated by equation (4), which in the shallow-water limit approaches equation (9). Evidence from Ou (1980) and the data in this report suggest that the coefficient α may not be a universal constant. There is also evidence that once very shallow depths are reached, other mechanisms can dominate spectral shape in the vicinity of the peak; the deviation, however, is such that equation (4) appears to be an overestimate.

IV. FORMULATION OF DEPTH-LIMITED SIGNIFICANT WAVE HEIGHT, H_L

Since equation (4) provides an estimate of the upper limit on energy density in water depth h as a function of frequency and wave generation condition as expressed by the coefficient α , it is possible to estimate the upper bound on the depth-limited wave energy, E_h , if a low-frequency cutoff value, f_c , is known. E_h can simply be estimated by

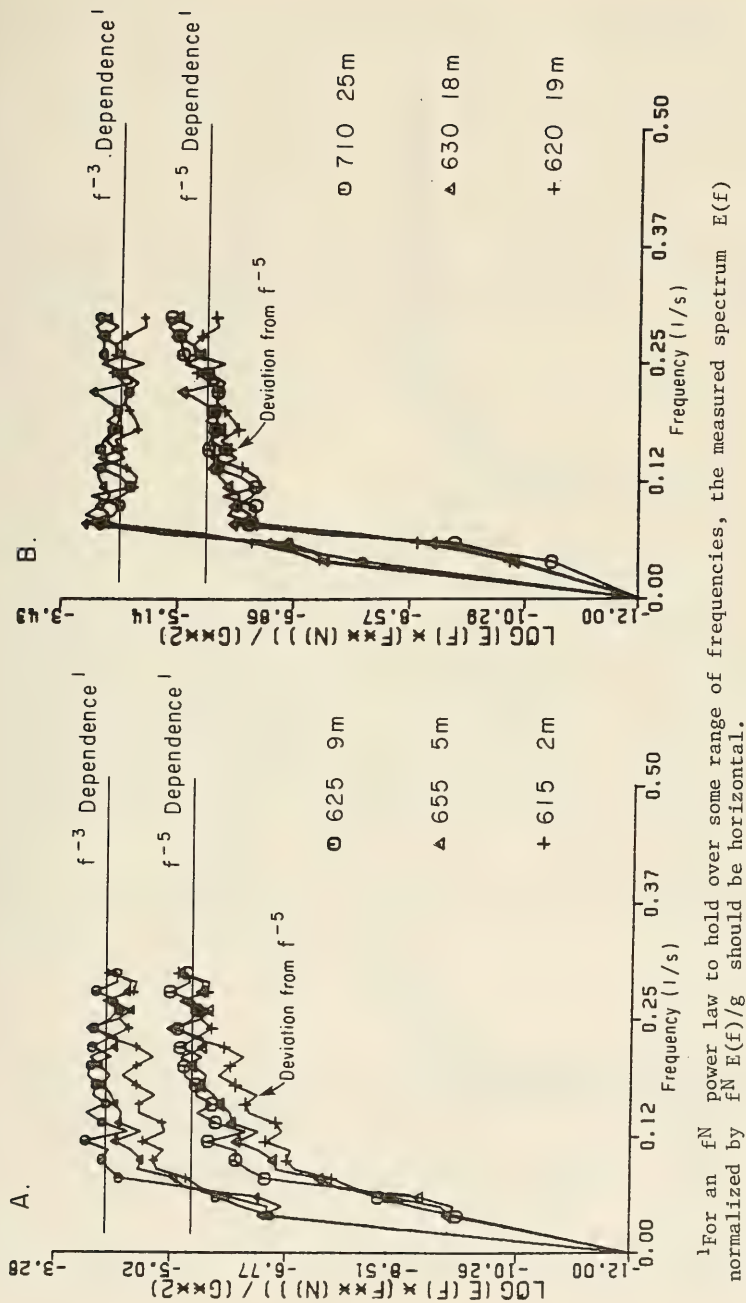


Figure 2. Comparisons of wave spectra at various depths to f^{-3} and f^{-5} laws. Wave spectra collected at Duck, North Carolina, on 25 October 1980 were normalized to display f^{-3} and f^{-5} dependencies. In graph A the deeper spectra are shown to follow a f^{-3} law over the frequency range from 0.1 to 0.2 hertz with the spectra deviating from this law in the 0.1- to 0.18-hertz range. In graph B the spectra tend to be more closely parallel to the f^{-3} law while significantly deviating from the f^{-5} law line.

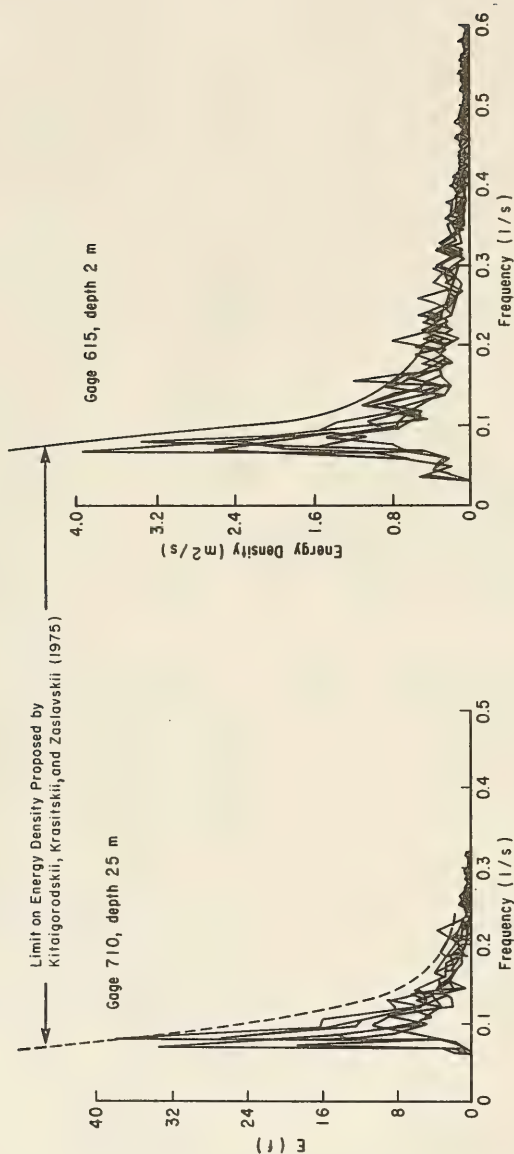


Figure 3. Selected storm spectra at different water depths. Spectra were selected to span the peak of the storm at different depths. $E_m(h,f)$ is plotted for each gage with a value of α of 0.0081 for the maximum depth that occurred during the time sequence. Note that the energy density scale for gage 615 is one-tenth that of gage 710.

$$E_h = \int_{f_c}^{\infty} E_m(f, h) df = \int_{f_c}^{\infty} \alpha g^2 f^{-5} \phi(\omega_h) / (2\pi)^4 df \quad (13)$$

The depth-limited significant wave height (spectral) is then

$$H_{\ell} = 4.0 (E_h)^{1/2} \quad (14)$$

In shallow water, H_{ℓ} is expected to be different from $H_{1/3}$, but how different is uncertain. Although $H_{1/3}$ has a long tradition of use in coastal engineering, the wave height H_{ℓ} defined in equation (14) appears to be a more consistent parameter because it is directly related to the energy of the wave field.

Figure 4 provides curves of H_{ℓ} as a function of cutoff frequency, f_c , and depth, h , for $\alpha = 0.0081$. If α is different an estimate of H_{ℓ} for that α can be made by

$$H_{\ell} = H_{\ell}^* (\alpha / 0.0081)^{1/2} \quad (15)$$

where H_{ℓ}^* is H_{ℓ} estimated with α of 0.0081.

Clearly the cutoff frequency and the value of α are crucial for obtaining estimates of H_{ℓ} . An examination of storm spectra indicates that the spectral peak is quite sharp. Consequently, a reasonable choice for f_c would be about 90 percent of f_p . If there is evidence of more energy on the forward face of the spectrum, f_c could be estimated by using a lower percentage. The parameter α can be obtained by fitting equation (4) to observed data if available. For field engineers, most often this may not be possible in which case α can be estimated by knowledge of the peak frequency, f_p , and windspeed, U , through the relationships developed by Hasselmann, et al. (1973). The values of f_p and U can be obtained from hindcasts or measurements. Figure 5 provides values of $(\alpha / 0.0081)^{1/2}$ as a function of f_p and U .

When the primary frequency components containing the major part of the energy are in shallow water, as determined by the condition $\omega_h < 1$, then E_m is given by equation (9). This can be integrated analytically to give an estimate of H for $\alpha = 0.0081$

$$H_{\ell} = \frac{1}{\pi} (\alpha g h)^{1/2} f_c^{-1} \quad (16)$$

Equation (16) has the remarkable consequence of suggesting that H_{ℓ} defined as $4.0(E)^{1/2}$ varies with the square root of depth when the primary spectral components are depth limited. The monochromatic depth-limited wave height, H_d , varies linearly with h .

V. FIELD AND LABORATORY EVIDENCE FOR DEPTH-LIMITED SIGNIFICANT HEIGHT, H_{ℓ}

In order to test the applicability of equations (15) and (16) in predicting H_{ℓ} in shallow water, laboratory data taken by Seelig and Broderick (1981) in

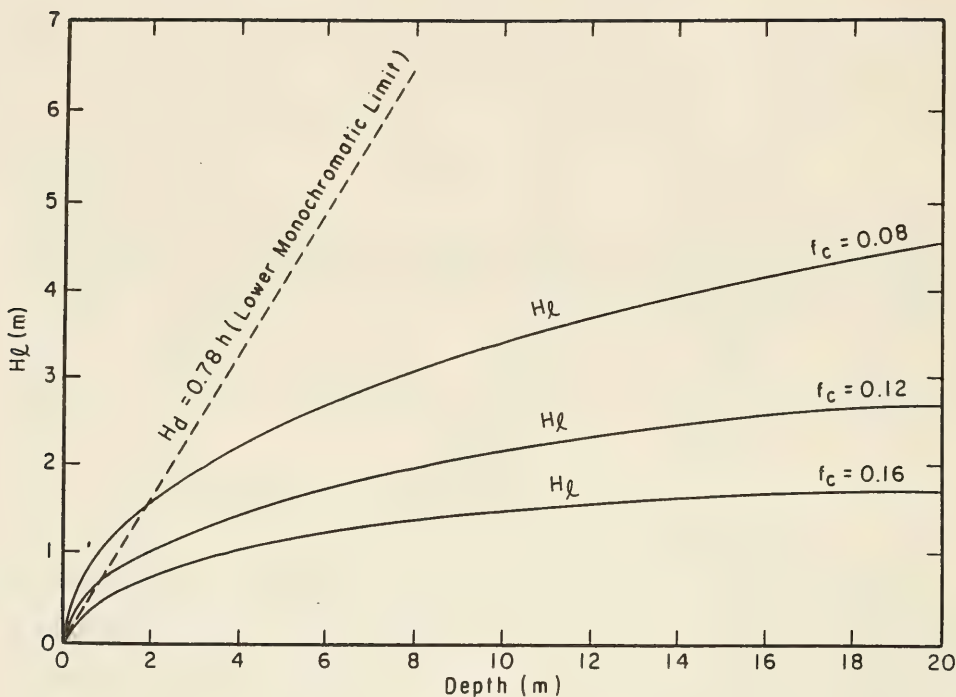


Figure 4. Depth-limited significant wave height, H_ℓ , as a function of water depth and cutoff frequency. Curves are calculated for $\alpha = 0.0081$. H_d is plotted for lower limit of $0.8 h$.

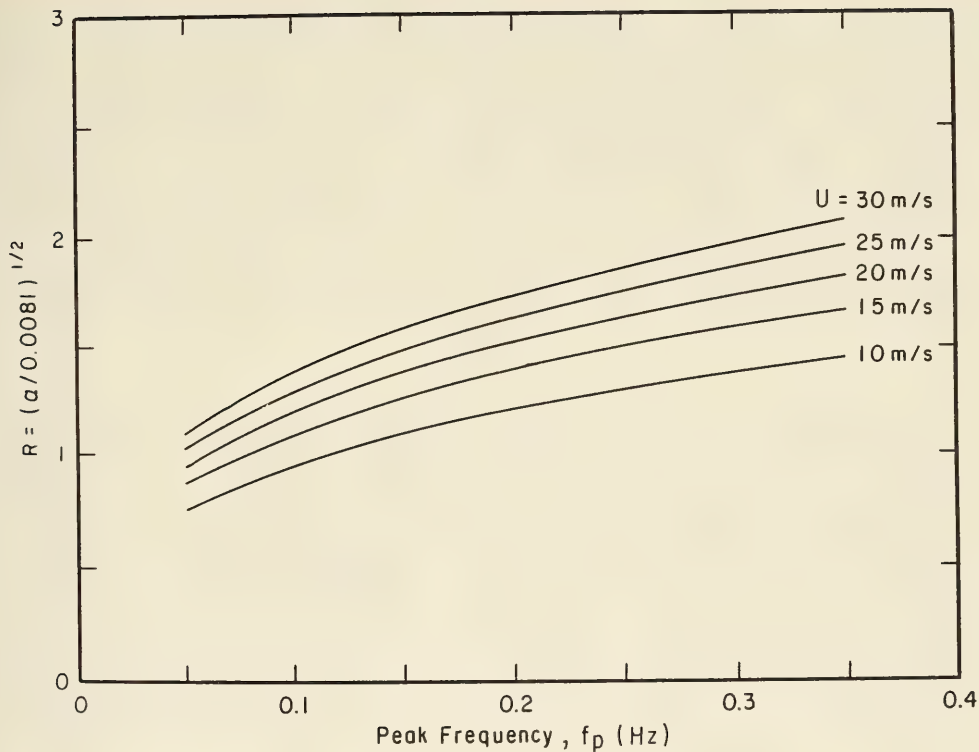


Figure 5. Plot of $R = (\alpha/0.0081)^{1/2}$ as function of peak frequency of spectrum and windspeed U . Data based on JONSWAP wind sea relationships (Hasselmann, et al., 1973). Coefficient R is used to adjust curves in Figure 4 to account for variation in α .

a flume 44 meters long and 0.45 meter wide with a maximum water depth of 0.6 meter and a bottom slope of 1:30 at one end were examined. Seelig and Broderick ran a variety of spectral shapes and energies. Figure 6 is a plot of H_ℓ calculated, as in equation (1), from a Fourier analysis of their wave data against $h^{1/2}$. Typically, the wave appear to shoal with decreasing depth, thereby increasing in height until a point is reached at which the wave height decreases linearly with the square root of depth. Figure 7 is an estimate of H_ℓ , based on equation (15), for two forms of f_c . A plot of the maximum individual wave, H_{max} , is plotted as is the monochromatic breaking limit which H_{max} appears to follow. H_ℓ is much less than the monochromatic breaking limit in this case. Figure 8 provides plots of H_ℓ versus $h^{1/2}$ for wave data at FRF on 25 October 1980. The value of h is estimated by an average of profiles before and after the storm and includes the tide and the wave setup. The curves are approximately linear with $h^{1/2}$.

VI. DISCUSSION

An examination of the characteristics of spectral shape in shallow water has led to a method of estimating the upper bound on wave energy as expressed by a depth-limited wave height. It is shown that in the shallow-water limit this leads to an approximate variation of H_ℓ with the square root of depth. Frequently, the monochromatic limiting value H_d is used to provide an upper bound on the wave height in shallow water. This report indicates that such an approach can significantly overestimate the significant wave height. The traditional method of estimating wave conditions in shallow water has been to obtain an estimate of $H_{1/3}$ in some depth of water, then refract and shoal it into the shore. At some point $H_{1/3}$ becomes larger than H_d , in which case $H_{1/3}$ is set to H_d . This report indicates, however, that the wave height H_ℓ , which is directly related to the wave energy, varies with $h^{1/2}$ and is normally much less than H_d . Consequently, when the energy in the sea is of concern, H_ℓ should be used rather than H_d . If the maximum individual wave that can occur is of concern then H_d is appropriate.

The method in this report also indicates that the maximum significant wave height, H_ℓ , in shallow water in lakes and bays can be different than that in the open ocean because the cutoff frequency, f_c , in the smaller water bodies is normally much higher than f_c for large ocean storms. Table 3 provides estimates for H_ℓ as a function of h for an ocean, a large lake, and a small lake for the same windspeed, U , of 25 meters per second but for different frequencies. Longer waves in an ocean are expected to develop than in small lakes; consequently, f_c is higher in the short fetch cases. The coefficient α increases in short fetch cases, but it enters H_ℓ through a square root relationship.

Estimates of depth-limited wave conditions have traditionally been based on linearity of wave height and depth. This linear relationship is well established for monochromatic waves by both laboratory and theoretical studies. Extensions to irregular wave conditions have relied on this linear relationship but with a coefficient of about 0.4. Figure 8 is a plot of this variation for 25 October 1980 and shows that in slope and magnitude this form is a poor predictor. The method in this report is based on a theory about spectral shape and appears to be a better predictor. It should be noted, however, that evaluations of the newer method must account for variations in α and f_c as wave conditions change. Hence, simply plotting H_ℓ versus h or $h^{1/2}$ for

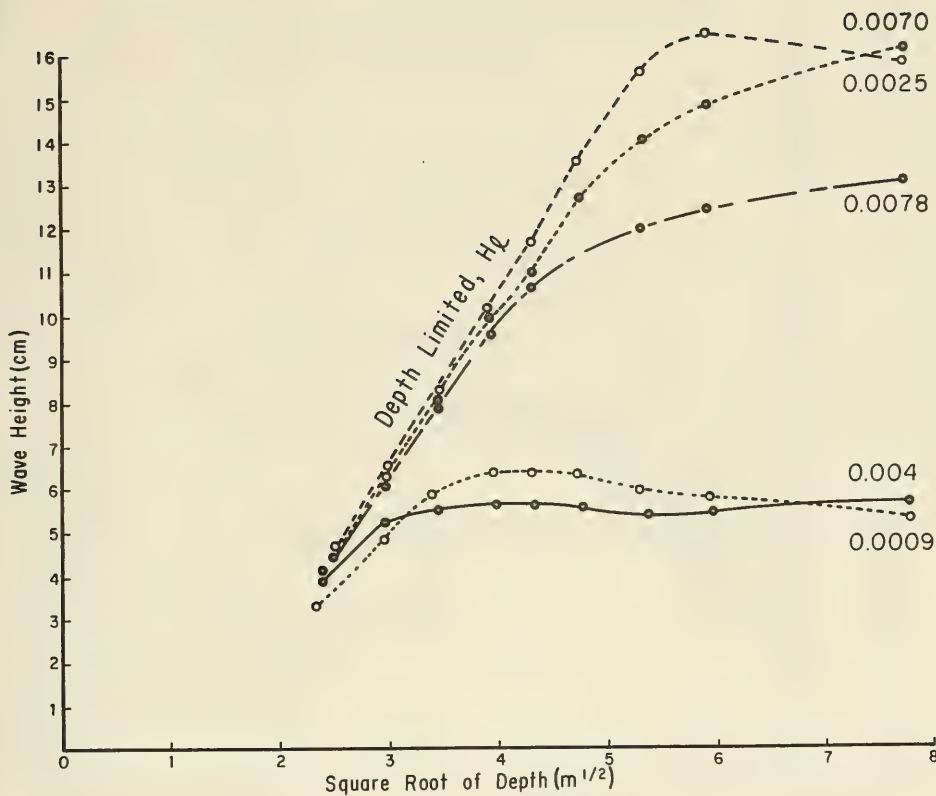


Figure 6. Variation of significant wave height, H_ℓ , with the square root of water depth. After a region of shoaling, wave height drops off linearly with the square root of water depth. Differing slopes are due to variations in α and f_c .

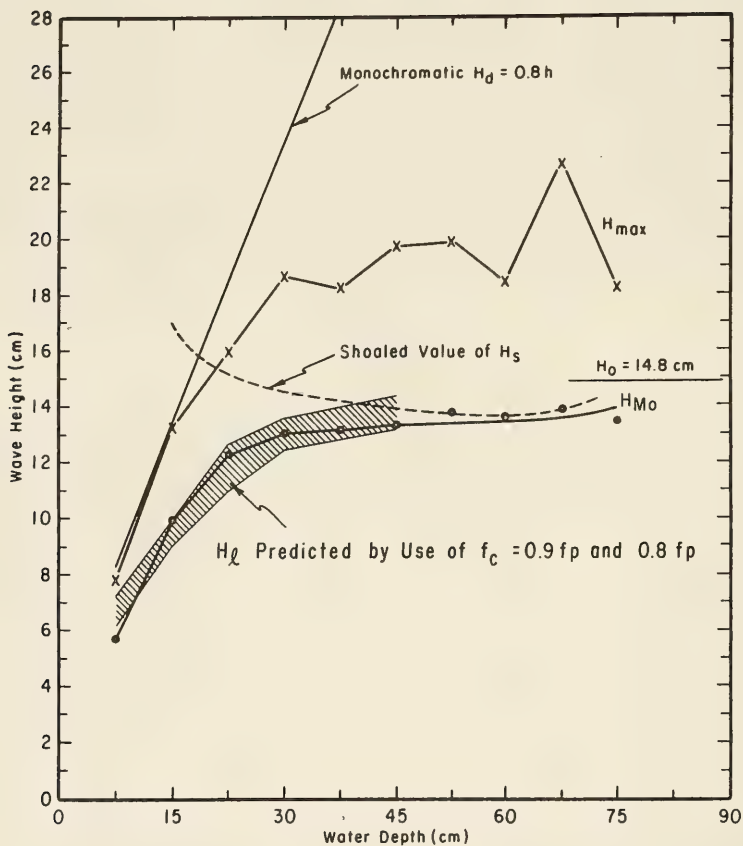


Figure 7. Estimate of H_l for laboratory conditions (from Seelig and Broderick, 1981). H_l is estimated using two estimates of f_c , and a line $H_d = 0.8 h$ is also provided. H_{Mo} at the toe of the 1:30 slope is 13.4 centimeters with $f_p^{-1} = 1.47$ seconds. A linear shoaling curve is also shown.

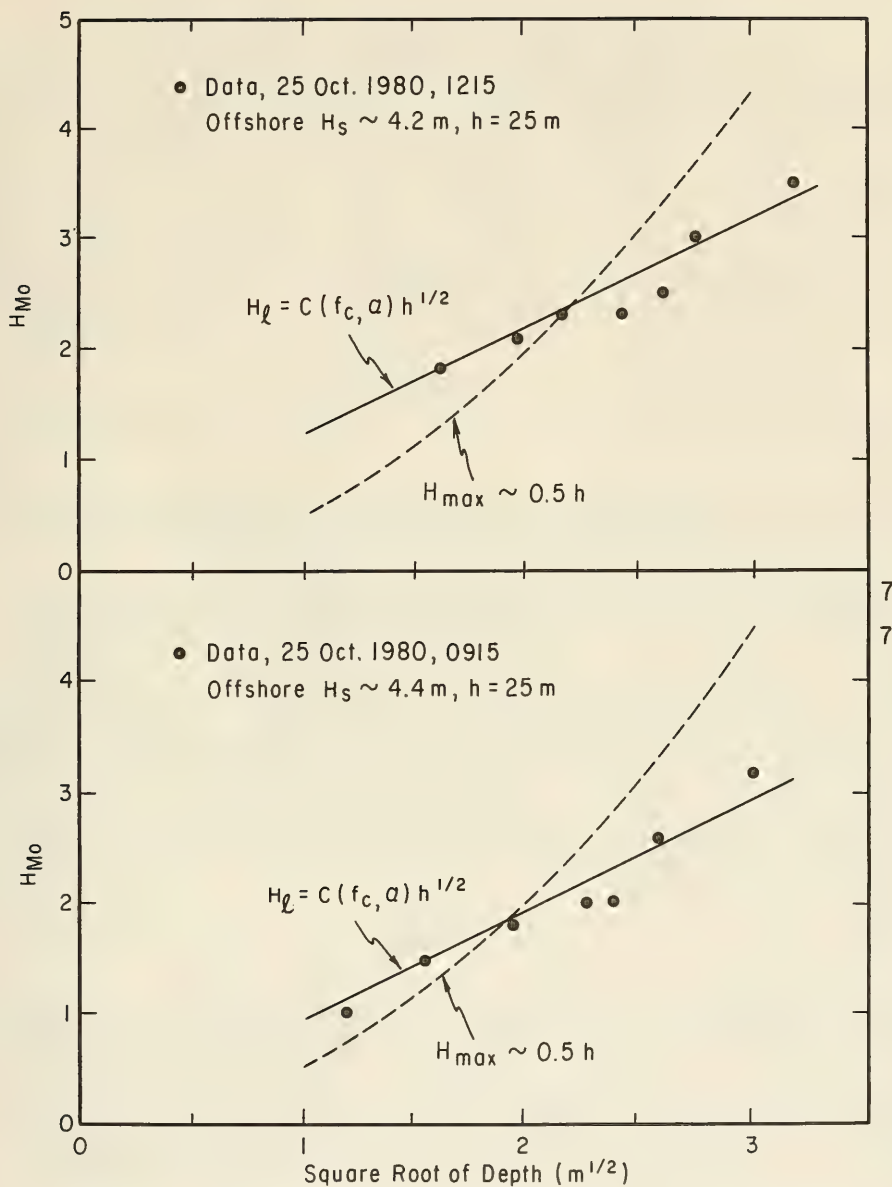


Figure 8. Variation of wave height with square root of depth, 25 October 1980, Duck, North Carolina. Solid line is based on measured α and f_c . Dashline represents estimated band on monochromatic theory with $H = 0.5 h$.

Table 3. Variation of H_p with depth for ocean, large lake, and small lake generation cases.

Depth (m)	Monochromatic wave, H_d ¹ (m)	Ocean H_ℓ ² (m)	Large lake, H_ℓ ³ (m)	Small lake, H_ℓ ⁴ (m)
1	0.78	1.5 ⁵	1.1 ⁵	0.9 ⁵
3	2.2	2.6 ⁵	1.9	1.6
5	3.9	3.4	2.5	1.9
8	6.2	4.2	3.0	2.3
10	7.8	4.7	3.3	2.6

¹Estimated by lower limit of 0.78 h.

² $f_p = 0.08$, $f_c = 0.07$, $(\alpha/0.0081)^{1/2} = 1.20$.

³ $f_p = 0.12$, $f_c = 0.11$, $(\alpha/0.0081)^{1/2} = 1.37$.

⁴ $f_p = 0.16$, $f_c = 0.14$, $(\alpha/0.0081)^{1/2} = 1.44$.

⁵Larger than H_d .

one gage will show considerable scatter because of the time variation of α and f_c . The evaluations of the method in this report have removed this constraint by using a series of gages across the nearshore zone.

The use of the method at the beginning of this report was restricted to spectra of some breadth such as storm seas. It is clear that nearly monochromatic waves follow the linear depth relationship, yet it is increasingly clear that irregular waves do not. A question of major importance not yet resolved is how wide must a spectrum be before the waves follow the relationships in this report. Equally important is the isolation of the physics of wave motion that determine these differences. In a shoaling monochromatic wave, nonlinearities arise which force the development of harmonics in the wave frequency and tend to broaden the spectrum, yet the absence of other wave components may reduce the transfer energies by resonant interactions. If the bottom slope is sufficiently steep, the evolution of the swell waves may be markedly different from irregular waves which may more easily exchange energy due to resonant interactions.

VII. SUMMARY

A method for estimating depth-limited significant wave height, H_ℓ , based on a theoretical form for the shape of shallow-water storm wave spectra was presented. The method requires an estimate of the peak frequency of the wave spectrum, f_p ; knowledge of the Phillips' equilibrium coefficient, α ; and water depth, h . A method for estimating α based on information about the peak frequency of the sea spectrum is also given. The results indicate that the depth-limited significant wave height, H_ℓ , based on the energy of the sea state is generally less than the depth-limited monochromatic wave height, H_d . The depth-limited wave height defined as $4.0(E)^{1/2}$ appears to be related to the square root of depth.

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